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Quantum Teleportation via a Two-Qubit Ising Heisenberg Chain with an Arbitrary Magnetic Field

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Abstract The quantum teleportation via a two-qubit Ising Heisenberg chain in the presence of an external magnetic field with an arbitrary direction are investigated. The effect of the orientation of an external magnetic field on the entanglement teleportation has been analyzed numerically. It is found that the teleported thermal concurrence and average fidelity can be maximized by rotating the magnetic field (with fixed magnitude) to an optimal direction. The ferromagnetic channel is not suitable to teleportation. A minimal entanglement of the thermal state is needed to realize the entanglement teleportation for antiferromagnetic channel. It is also found that the entanglement of the channel cannot completely reflect the teleported concurrence and average fidelity. There exist double-value phenomena between them.

Keywords Quantum teleportation · Two-qubit Ising Heisenberg chain · Arbitrary magnetic field

1 Introduction

Entanglement, one of the most fascinating quantum mechanical features, can constitute a significant resource for quantum information processing [1, 2], such as quantum teleportation [3, 4], superdense coding [5], quantum cryptographic key distribution [6], and so on. Recently, quantum spin chain in condensed matter physics has attracted much attention on characterizing entanglement and its teleportation [7–13]. The quantum teleportation can be accomplished successfully through a two-qubit Heisenberg model, with fidelity better than any classical communication protocol. The effects of anisotropy and magnetic field on quantum teleportation via a two-qubit Heisenberg XY chain were investigated by Yeo et al. [14] and a two-qubit Heisenberg XXZ chain by Zhou and Zhang [15]. The influences of spin-orbit interaction on the entanglement teleportation were discussed for a two-qubit Heisenberg XXX chain in the absence of a magnetic field [16] and a two-qubit Heisenberg

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XYZ chain in an inhomogeneous magnetic field [17], respectively. Influence of intrinsic decoherence on quantum teleportation via two-qubit Heisenberg XYZ chain in an external uniform magnetic field was studied [18]. Enhanced teleportation through two-spin anisotropic XYZ Heisenberg quantum spin chains was analyzed [19]. The partial teleportation of entanglement through natural thermal entanglement in two-qubit Heisenberg XXX Chain was discussed [20]. A scheme of teleporting an unknown state via the generalized two-mode binomial states was proposed, and the mean fidelity of the scheme calculated [21]. Teleportation of an Arbitrary Two-Particle State via a Single Cluster-Class State was studied [22]. However, the entanglement teleportation for a two-qubit Heisenberg chain under an arbitrary direction magnetic field has rarely been analyzed.

The one-dimension Ising model describing a set of linearly arranged spins is absent of entanglement completely. However, a component of an external magnetic field, along the direction perpendicular to the Ising orientation, no matter how small, is enough to produce entanglement [23] in antiferromagnetic case. Therefore the quantum teleportation, through the thermally entangled state of two-qubit Heisenberg Ising chain, with fidelity better than any classical communication protocol, is possible. In this paper, the quantum teleportation via a two-qubit Ising spin chain in the presence of an external magnetic field with arbitrary direction has been investigated. The numerical results for teleported concurrence and average fidelity have been obtained. The effects of direction of an external magnetic field on the teleported concurrence and average fidelity have been discussed. In addition, the relation between entanglement and teleported concurrence and average fidelity has been analyzed.

This paper is organized as follows. In the forthcoming section the Hamiltonian of the system is introduced, and the corresponding thermal density matrix is obtained. The entanglement teleportation at the ground state and excited states are discussed in Sect. 3. In Sect. 4, the fidelity of teleportation through Ising chain is investigated. Finally, the conclusions are drawn in Sect. 5.

2 The Hamiltonian and Thermal Density Matrix

The Hamiltonian H of a two-qubit Heisenberg Ising chain under an external magnetic field B along arbitrary direction is given by [23]

$$H = 2J\sigma_1^z\sigma_2^z + B\sin\alpha\big(\sigma_1^x + \sigma_2^x\big) + B\cos\alpha\big(\sigma_1^z + \sigma_2^z\big), \tag{1}$$

where σ_i^α $(\alpha = x, z)$ is the Pauli matrices at sites $i = 1, 2$. The chain is called to be antiferromagnetic for $J > 0$ and ferromagnetic for $J < 0$. α $(0 \le \alpha \le \pi)$ is the angle between the direction of external magnetic field and the Ising orientation. It is reasonable to confine the variation of B $(B \ge 0)$ within the plane containing the Ising orientation, because the Hamiltonian possesses rotational symmetry about the z axis in 3 spatial dimensions.

In the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, after straightforward calculation, the eigenstates and the corresponding eigenvalues of Hamiltonian (1) are found to be:

$$\begin{aligned}
|\psi^i\rangle &= \frac{1}{N_i}\big(2B_x|00\rangle + (\varepsilon_i - 2J + 2B_z)|01\rangle + (\varepsilon_i - 2J + 2B_z)|10\rangle \\
&\quad + \big[(\varepsilon_i + 2J)(\varepsilon_i - 2J + 2B_z) - 2B_x^2\big]|11\rangle\big) \quad (i = 1, 2, 3), \\
|\psi^4\rangle &= \frac{1}{\sqrt{2}}\big(|01\rangle - |10\rangle\big),
\end{aligned} \tag{2}$$

and

$$\varepsilon_i = 2\sqrt{-\frac{p}{3}}\cos\frac{\delta+(i-1)2\pi}{3}+\frac{2J}{3} \quad (i=1,2,3)$$
$$\varepsilon_4 = -2J, \tag{3}$$

where $B_x = B\sin\alpha$, $B_z = B\cos\alpha$, $p = -4(\frac{4J^2}{3}+B^2)$, $q = -\frac{16}{27}J^3+\frac{8J}{3}(2J^2+2B_x^2-4B_z^2)$, $\delta = \arccos(-\frac{q}{2\sqrt{-(p/3)^3}})$, $\xi = \cos\alpha$ and N_i $(i=1,2,3)$ is the normalization.

Specially reminded, ξ $(0 \le \xi \le 1)$ is regarded as the direction parameter for expressing the effects of direction of an external magnetic field. Thus hereafter we will use ξ rather than the original parameter α in our discussion. The value $\xi = 1$ corresponds to the case of external magnetic field parallel to z axis. In contrast, the value $\xi = 0$ corresponds to the case of external magnetic field perpendicular to z axis. ξ ranges continuously from 0 to 1 as the direction of an external magnetic field varies.

At the thermal equilibrium, the density matrix of the above system is described by

$$\rho(T) = \frac{1}{Z}\left[e^{-\beta\varepsilon_1}|\psi^1\rangle\langle\psi^1| + e^{-\beta\varepsilon_2}|\psi^2\rangle\langle\psi^2| + e^{-\beta\varepsilon_3}|\psi^3\rangle\langle\psi^3| + e^{-\beta\varepsilon_4}|\psi^4\rangle\langle\psi^4|\right], \tag{4}$$

where the partition function $Z = e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2} + e^{-\beta\varepsilon_3} + e^{-\beta\varepsilon_4}$, the Boltzmann's constant $k \equiv 1$ for simplicity from hereon, and $\beta = 1/T$.

3 Entanglement Teleportation

As it is well known, entanglement teleportation is to teleport an entangled state. In this section, the standard teleportation protocol [24] is considered by making use of the above thermal mixed state of two-qubit Ising chain as a resource. Without loss of generality, suppose the unknown arbitrary pure state to be teleported has the form

$$|\psi\rangle_{in} = \cos\frac{\theta}{2}|10\rangle + e^{i\phi}\sin\frac{\theta}{2}|01\rangle, \tag{5}$$

where $0 \le \phi \le 2\pi$ is the phase difference between the two bases and $0 \le \theta \le \pi$ describes all states with different amplitudes. The density matrix related to $|\psi\rangle_{in}$ is in the form

$$\rho_{in} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sin^2(\theta/2) & e^{i\phi}\sin\theta & 0 \\ 0 & e^{-i\phi}\sin\theta & \cos^2(\theta/2) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

Therefore the concurrence of the initial state is $C_{in} = 2|\sin(\theta/2)\cos(\theta/2)e^{i\phi}|$. The output state is given by [24]

$$\rho_{out} = \sum_{ij} p_{ij}(\sigma_i \otimes \sigma_j)\rho_{in}(\sigma_i \otimes \sigma_j), \tag{7}$$

where σ_i $(i = 0, x, y, z)$ stand for the unit matrix and three components of the Pauli matrix $\vec{\sigma}$, respectively, and $p_{ij} = tr[E^i\rho(T)]tr[E^j\rho(T)]$, in which $E^0 = |\Psi^-\rangle\langle\Psi^-|$, $E^1 = |\Phi^-\rangle\langle\Phi^-|$, $E^2 = |\Phi^+\rangle\langle\Phi^+|$, $E^3 = |\Psi^+\rangle\langle\Psi^+|$, here $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$, and $|\Phi^\pm\rangle =$

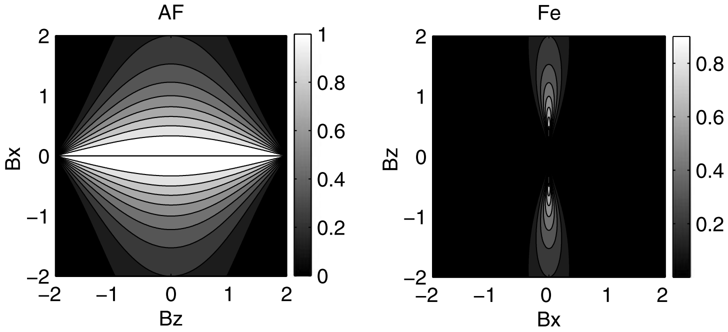

Fig. 1 (Color online) Contour plots of the teleported concurrence C_{out} at zero temperature in a Cartesian coordinate system. The *left* and *right panels* correspond to the antiferromagnetic case ($J = 1$) and ferromagnetic case ($J = -1$), respectively. $C_{in} = 1$. All the parameters are dimensionless

$(|00\rangle \pm |11\rangle)/\sqrt{2}$ are the Bell states. ρ_{in} signifies the density matrix of the input state. The concurrence of the output state can be determined by the positive square roots λ_i of the eigenvalues of the non-Hermitian spin-flipped matrix operator $R_{out} = \rho_{out}(\sigma_y \otimes \sigma_y)\rho^*_{out}(\sigma_y \otimes \sigma_y)$, the asterisk '$*$' indicates the complex conjugation. Thus, the concurrence of the output state can be expressed as

$$C_{out}(\rho_{out}) = \max\left\{0, 2\lambda_{\max} - \sum_{i=1}^{4} \lambda_i\right\}, \tag{8}$$

where

$$\begin{aligned}
\lambda_{1,2} &= (\mu + \nu)(\omega + x) \pm \left|(y + y^*)(z + z^*)(f + f^*)\right|, \\
\lambda_{3,4} &= \sqrt{\left[(\omega + x)^2 g + (\mu + \nu)^2 h\right]\left[(\mu + \nu)^2 g + (\omega + x)^2 h\right]} \\
&\quad \pm \left|(z + z^*)^2 f + (y + y^*)^2 f^*\right|,
\end{aligned} \tag{9}$$

with $\mu = \langle 00|\rho|00\rangle$, $\nu = \langle 11|\rho|11\rangle$, $\omega = \langle 01|\rho|01\rangle$, $x = \langle 10|\rho|10\rangle$, $y = \langle 01|\rho|01\rangle$, $z = \langle 01|\rho|10\rangle$ and $g = \frac{1}{2}(1 - \cos\theta)$, $h = \frac{1}{2}(1 - \sin\theta)$, $f = \frac{1}{2}e^{i\phi}\sin\theta$, the asterisk '$*$' indicates the complex conjugation. $C_{out}(\rho_{out})$ is dependent on the entanglement of the input state and the parameters of the channel, which is nonzero only for a particular choice of the channel parameters.

Although the expression for the concurrence $C_{out}(\rho_{out})$ can be solvable analytically, the expression is long-winded and complicated, so $C_{out}(\rho_{out})$ is present in graphical form (Figs. 1–6). Firstly, at zero temperature, Fig. 1 shows the contour plots of the teleported thermal concurrence C_{out} in a Cartesian coordinate system. It is easy to find that the entanglement of the output state through antiferromagnetic channel is different from that through ferromagnetic channel, which is in agreement with that of quantum teleportation via three-qubit Heisenberg XX ring under an external uniform magnetic field along the z axis [25]. The teleportated entanglement C_{out} is symmetric about $B_z = 0$. For antiferromagnetic channel, the region around $B_x = 0$ with $|B_z| < 2J$ has the highest possible teleported entanglement. There is no teleportated entanglement of the channel when B_x equals zero exactly. Furthermore, the teleported entanglement decreases with the increasing of B_x. This is due to the fact that the entanglement of the channel decreases when the component B_x is in-

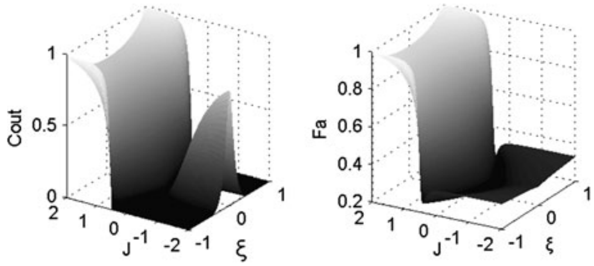

Fig. 2 At zero temperature, the teleported concurrence C_{out} (in the *left panel*) and the teleportation fidelity *Fa* (in the *right panel*) vs the coupling constant J and the direction parameter ξ. $B = 1$. $C_{in} = 1$. All the parameters are dimensionless

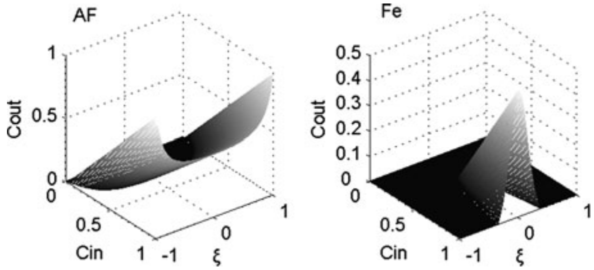

Fig. 3 (Color online) The teleported concurrence C_{out} as a function of the input concurrence C_{in} and the direction parameter ξ at zero temperature. $B = 1$. The *left* and *right panels* are for antiferromagnetic case and ferromagnetic case, respectively. All the parameters are dimensionless

creased. In contrast, for ferromagnetic channel, the region around $B_x = 0$ with $B_z = 0$ has the highest possible teleported entanglement. Here, the maximal teleported entanglement line around $B_x = 0$ is so thin that it is invisible. It is excited that rotating the magnetic field very slightly from the z direction can improve the teleported entanglement C_{out} rapidly no matter teleported channel is antiferromagnetic or ferromagnetic, i.e., varying the direction of the magnetic field can develop teleported entanglement.

In order to verify above explanation more clearly, the teleported thermal concurrence C_{out} vs the coupling constant J and the direction parameter ξ is shown as the left panel of Fig. 2. Obviously, C_{out} behaves differently for $J > 0$ and $J < 0$. For the antiferromagnetic channel, in the region around $\xi = 1$ or 0 with $J > 0.5B$, the teleported entanglement C_{out} has the highest possible value. While for the ferromagnetic channel, only at the region around $\xi = 0$ is the teleported entanglement C_{out} nonvanishing when the initial state is a maximum entangled state for giving coupling constant J.

The teleported thermal concurrence C_{out} as a function of the input concurrence C_{in} and the direction parameter ξ is plotted in Fig. 3. For the antiferromagnetic channel, the teleported thermal concurrence C_{out} increases linearly as C_{in} increases, which is agreement with Ref. [26], but the varying rate of this enhancement is determined by the direction parameter ξ. However, in the case of ferromagnetic channel, C_{out} is zero for small values of C_{in}. As C_{in} crosses a threshold value, C_{out} increases with a rate determined by the direction parameter ξ. From Fig. 3, it is significant that the rate of teleported entanglement C_{out} increasing with input concurrence C_{in} depends on the direction parameter ξ no matter teleported channel is antiferromagnetic or ferromagnetic.

Next, in the case of finite temperature, for the antiferromagnetic channel, a numerical solution of the teleported thermal concurrence C_{out} is shown as Fig. 4. With the increasing of temperature, the maximal teleported thermal concurrence C_{out} decreases since the entanglement of the channel decreases with temperature. Compared with Fig. 1, it also tells us that the line of zero concurrence C_{out} at $B_x = 0$ for $T = 0$ has broadened into a region of almost zero entanglement, and C_{out} drops to zero fast with the increasing of the $|B_x|$ component. Thus the maximal concurrence C_{out} reaches at some intermediate value of $|B_x|$ with $B_z = 0$.

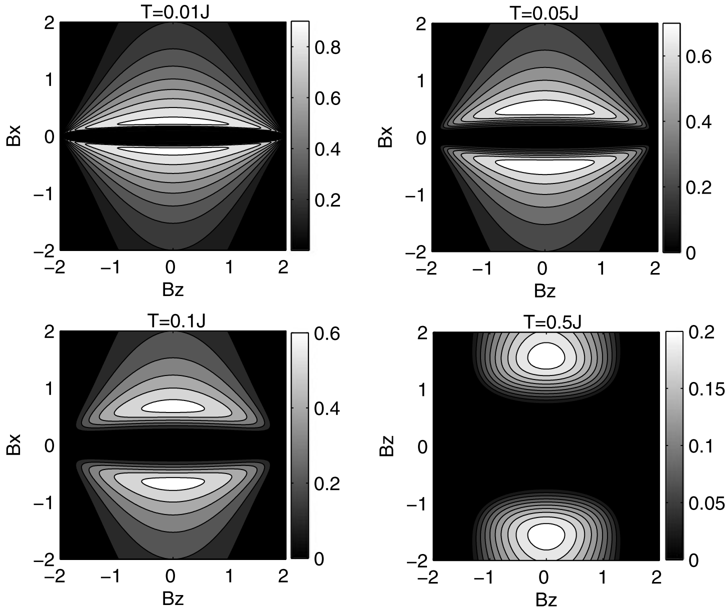

Fig. 4 Contour plots of the teleported thermal concurrence C_{out} at various finite temperatures for antiferromagnetic case in a Cartesian coordinate system. $J = 1$. $C_{in} = 1$. All the parameters are dimensionless

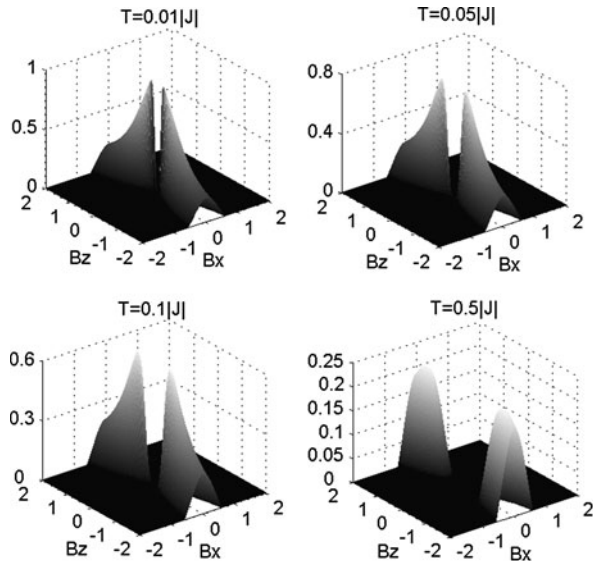

Fig. 5 (Color online) A plots of the teleported thermal concurrence C_{out} vs the component B_x and B_z of B at various finite temperatures for ferromagnetic case. $J = -1$. $C_{in} = 1$. All the parameters are dimensionless

As far as the ferromagnetic channel is concerned, Fig. 5 shows a numerical solution when the teleported thermal concurrence C_{out} is plotted as a function of components B_x and B_z. It told us that the slight departure of magnetic field from Ising direction leads to a rapid rise in teleported entanglement, i.e., there occurs variety sharply at $B_x = 0$. It is also noticed

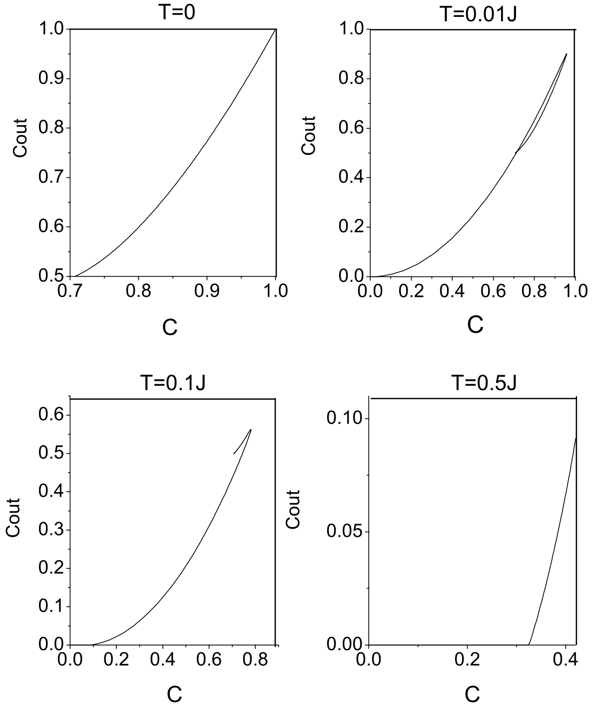

Fig. 6 The teleported thermal concurrence C_{out} vs concurrence C for antiferromagnetic case $J = 1$. $B = 1$. $C_{in} = 1$. The figures are plotted for varied direction parameter at different temperatures. All the parameters are dimensionless

that the increase of temperature widens the zero concurrence C_{out} zone around $B_x = 0$, and depresses the maximum of concurrence C_{out}.

Compared the teleported concurrence C_{out} (shown as Figs. 1–2) with the tangle τ in Ref. [23], it is found that the teleported concurrence C_{out} is greater if given more amount of concurrence C of the channel in the ground state. So, we turn our attention to the relation between teleported concurrence C_{out} and the concurrence C of the channel in general situation. The teleported thermal concurrence C_{out} versus concurrence C of the channel for antiferromagnetic case is plotted in Fig. 6. It is reasonable for practical case to assume that only the direction parameter ξ can be controlled for a given antiferromagnetic chain. At zero temperature, C_{out} is the monotone increasing function of C. There exists threshold value of concurrence C for entanglement teleportation at finite temperature. It is noted that the quantum channel needs the minimal entanglement to realize the entanglement teleportation of $C_{out} > 0$, which is no difficult to understand. In general, owing to the entanglement cannot be increased under local operations, the teleported entanglement decreases via the quantum channel. So the entanglement after the teleportation is always less than that of the input state. Furthermore, it is interesting that C_{out} is the monotone increasing function of C when the C is small at lower temperature (for example $T = 0.01, 0.1$), and increasing C gives two different C_{out}s. It is to say that there exists double-value phenomena between them. In fact, this result can be understood easily. In order to efficiently teleport a quantum state, one can control the temperature and the direction parameter. A given concurrence C corresponds to different temperatures and direction parameter, which give different teleported concurrence C_{out}.

4 The Fidelity of Entanglement Teleportation

In the following discussion, we turn our attention to the quality of the entanglement teleportation. In order to characterize the quality of the teleported state ρ_{out}, the concept of fidelity is usually introduced, which indicate the teleportation performance of a quantum channel when the input state is a pure state [27]:

$$F(\rho_{in}, \rho_{out}) = \left[tr\sqrt{(\rho_{in})^{1/2}\rho_{out}(\rho_{in})^{1/2}} \right]^2. \tag{10}$$

The average fidelity Fa of teleportation can be formulated as

$$Fa = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} F \sin\theta d\theta}{4\pi}. \tag{11}$$

After some straightforward algebra, the amount of average fidelity is

$$Fa = \frac{1}{3}(\mu+\nu)^2 + \frac{2}{3}(\omega+x)^2 + \frac{1}{3}(z+z^*)^2, \tag{12}$$

where μ, ν, ω, x, z, and '$*$' are the same as ones in (9).

The average fidelity Fa is dependent on the channel parameters. In order to transmit $|\psi\rangle_{in}$ with better fidelity than any classical communication protocol, Fa must be greater than $2/3$. From the right panel of Fig. 2, it is easy to find that only antiferromagnetic chain with stronger coupling is suitable for using quantum teleportation channel, while ferromagnetic chain is not (though the teleported entanglement of the channel can exist by adjusting the direction of magnetic field). This is in agreement with quantum teleportation through a two-qubit Heisenberg XXZ chain [28], but disagrees with Ref. [25]. Figure 7 gives plot of the average fidelity Fa at various finite temperatures for antiferromagnetic channel. The behavior is similar to that of concurrence C_{out} (see Figs. 1 and 4). Here, the explanation does not repeat any more. However, the average fidelity Fa has the value of $2/3$, which is the limited value in classical communication, when B_x equals zero exactly, substituting for zero.

In order to clearly show the relation between the average fidelity Fa and the concurrence C, the average fidelity Fa versus a concurrence C is plotted in Fig. 8, where it is assumed that only the direction of magnetic field and temperature can be controlled in antiferromagnetic case. The figure (a) is plotted for varied temperatures at different direction parameter. The figure (b) is plotted for varied direction parameter at different temperature. Compared with Fig. 6, it is not difficult to find that the average fidelity Fa exhibits analogous characteristic to teleported thermal concurrence C_{out}. In fact, the behavior of Fa is similar to that of C_{out}, except that it also depends on the input states. From Fig. 8(b), we have found even more interesting character. At finite temperature, the lowerer the temperature is, the broader the double-value zone of concurrence C is. The double-value phenomena told us that the concurrence of channel cannot completely reflect the average fidelity. In fact, it can be understood easily from Figs. 2 and 6 in Ref. [23] (where θ corresponds to ξ here). Since a given entanglement corresponds to infinite different direction of magnetic fields and temperature (ξ, T), those (ξ, T) give infinite average fidelity. The maximal Fa can be obtained by maximizing the function $Fa(\xi, T)$ under the constraint of fixed entanglement. There is a monofunctional relation between the average fidelity Fa and the concurrence C when the direction parameter equals one or exceeds threshold value.

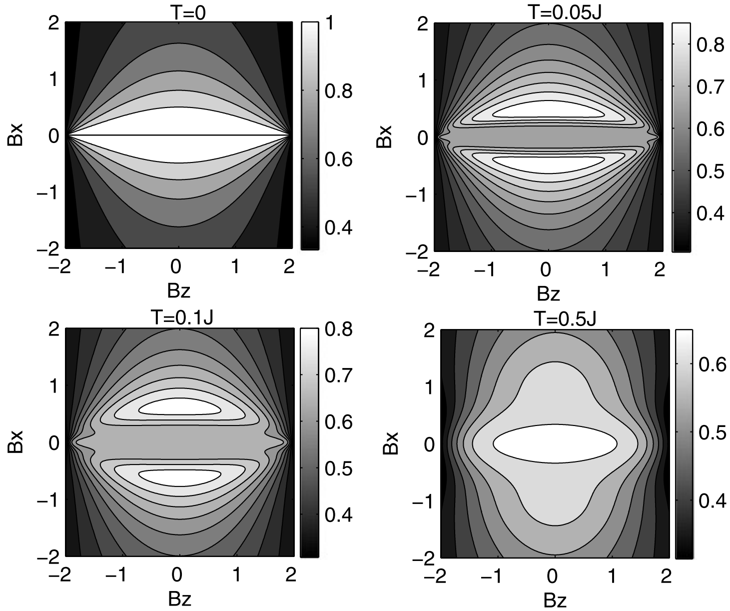

Fig. 7 Contour plots of the teleportation fidelity *Fa* at various finite temperatures for antiferromagnetic case in a Cartesian coordinate system. $J = 1$. $C_{in} = 1$. All the parameters are dimensionless

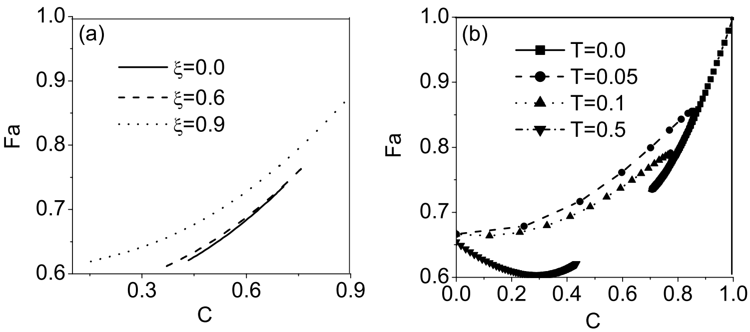

Fig. 8 The teleportation fidelity *Fa* vs concurrence *C* for antiferromagnetic case $J = 1$. $C_{in} = 1$. $B = 1.0$. The figure (**a**) is plotted for varied temperatures at different direction parameter ξ, from the top to bottom, the *dot line* is for 0.9, the *dash line* for 0.6, the *solid line* for 0.0, respectively. The figure (**b**) is plotted for varied direction parameter ξ at different temperature, the *solid line with squares* is for 0.0, the *dash line with circles* for 0.05, the *dot line with triangle* for 0.1, the *dash and dot line* with reversed-triangle for 0.5, respectively. All the parameters are dimensionless

5 Conclusion

In this paper, the quantum teleportation via a two-qubit Ising spin chain in the presence of arbitrary direction external magnetic field has been investigated. The effect of the orientation of an external magnetic field on the entanglement teleportation has been analyzed numerically. The teleported thermal concurrence and average fidelity can reach a maximum value by adjusting the direction of an external magnetic field. The average fidelity is always less than classical limit $2/3$, no matter what the direction of an external magnetic field for two-qubit ferromagnetic Ising channel. In the case of two-qubit antiferromagnetic Ising channel, a minimal entanglement of the thermal state is needed to realize the entanglement teleportation, and the teleported concurrence and average fidelity simultaneously reach their maximal values. Adjusting the magnetic field very slightly from the z direction can rapidly improve the teleported entanglement C_{out} whether teleported channel is antiferromagnetic or ferromagnetic, i.e., changing the direction of the magnetic field can develop teleported entanglement. It is interesting that the entanglement of the channel cannot completely reflect the teleported concurrence and average fidelity, larger amount of entanglement does not mean better teleportation, for a fixed entanglement corresponds to infinite teleported concurrences or average fidelities.

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